

CS4770 Pattern Recognition Systems Monsoon 2006

Home Assignment 1. Due: August 18, 2006

1. A triangle probability density function $p(x) \sim T(\mu, \sigma)$ has its peak value at μ and falls off linearly to zero at $\mu \pm \sigma$.
 - (a) Compute the probability $p(\mu)$.
 - (b) Given a 3-category classification problem with priors: $P(\omega_1) = 0.2$, $P(\omega_2) = 0.3$, and $P(\omega_3) = 0.5$. The likelihoods are: $p(x|\omega_1) \sim T(1, 2)$, $p(x|\omega_2) = T(4, 3)$, and $p(x|\omega_3) = T(6, 5)$.

Plot the likelihood functions for all 3 classes.
 - (c) Plot the posterior probabilities (ignore scale) for all 3 classes. Find the regions of x which get classified as ω_1, ω_2 , and ω_3 .
 - (d) Classify the following values of x using Bayes' rule into appropriate categories: 1.0, 2.0, 3.0, 4.0, 5.0.
2. A farm has 60% cows and 40% goats. Goats have a length given by $N(1, 0.4)$ and cows have a length given by $N(2, 0.8)$, where $N(\mu, \sigma)$ denotes a normal distribution of mean μ and variance σ^2 . It is required to classify an animal as a cow or a goat based on their measured length using a Bayesian classifier.
 - (a) Draw the distributions and show the threshold on length that separate cows and goats. Indicate the region of error in the figure.
 - (b) What labels will be assigned to animals whose lengths are measured to be 1.2, 1.4, 1.6, and 1.8?
 - (c) Based on the classification generated by your algorithm, cows are fed cow-food and goats are fed goat-food. Cows and goats get sick if fed with the wrong food. If this happens, the medicine for the cows cost Rs 10 and for the goats cost Rs 40.

What labels will be assigned to animals whose lengths are measured to be 1.2, 1.4, 1.6, and 1.8 if the objective is to minimize the spending on the medicines?
3. Another farm has 60% cows and 40% goats but classifies animals based on the length and weight. Goats have a distribution given by $N([1, 10]^T, \Sigma)$ and cows by $N([2, 15]^T, \Sigma)$, where $\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 0.4 & 0.5 \\ 0.5 & 2.0 \end{bmatrix}$.
 - (a) What is the decision boundary between the goat and cow classes using Bayesian classification? Give the analytical expression of the boundary. Plot it approximately in the 2D space.
 - (b) What labels will be assigned to animals with the following measurements?
 - (i) $[1.2 \ 14]^T$, (ii) $[0.9 \ 16]^T$, (iii) $[1.8 \ 12]^T$, and (iv) $[1.9 \ 19]^T$.
 - (c) Repeat (a) and (b) above with the off-diagonal elements $\sigma_{12} = 0$.
 - (d) Write the analytical expression for the Bayesian decision boundary if the the cow class had a covariance matrix with $\sigma_{12} = 0.8$ and $\sigma_{22} = 3.0$. (Goat class has the original Σ .) Intuitively describe this boundary.