Morphological Image Processing

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Morphological Operations

- Neighbourhood operations carried out in spatial domain
- Based on mathematical morphology
  - set theoretical framework
  - originally for binary images
  - extended for grey scale images
- Applications:
  - extract info about forms and structures
  - shaping and filtering of forms and structures
Morphological Processing

• Consists essentially of two steps:
  • Probe a given object in x[m,n] with a structuring element (se)
  • Find how the se fits with the object
• Information about fit is used to
  • extract info about the form of object; OR
  • change pixel values and shape objects
• Different size & shape of se yields different kinds of info about the object; shapes the regions in different ways
Set Theory Basics

- Union, Intersection
- Complement, Difference
- Subset, Superset, Disjoint Sets

- Reflection: \( \hat{B} = \{ w \mid w = -b, \text{ for } b \in B \} \)

- Translation: \( (A)_z = \{ c \mid c = a + z, \text{ for } a \in A \} \)
Examples

(A) \[ z = (3,5) \]
Morphological Operations/Algorithms

- Basic Morphological Operations
  - Dilation
  - Erosion
  - Opening
  - Closing
  - Hit-or-Miss Transformation

- Morphological Algorithms

- Extensions to Grayscale
Dilation

- Dilation of $A$ by $B$: $A \oplus B$

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \Phi\} \quad A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$
Dilation: Example

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Dilation: Example

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

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Erosion

- Erosion of $A$ by $B$: $A \ominus B$

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}.$$
Erosion: Example
Erosion + Dilation
Opening and Closing

- Opening: Erosion followed by Dilation
- Opening $A$ by $B$: $A \circ B = (A \Theta B) \oplus B$
  - Smoothes Contours, Breaks narrow bridges, Eliminates thin protrusions

- Closing: Dilation followed by Erosion
- Closing $A$ by $B$: $A \bullet B = (A \oplus B) \Theta B$
  - Smoothing, Closes small holes and channels
Opening: Physical Interpretation

\[ A \cdot B = \bigcup \{(B)_z | (B)_z \subseteq A\} \]

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Closing: Physical Interpretation
Opening: Example

Erosion

Dilation

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Closing: Example

Dilation

Erosion

\[ A \oplus B = \{ A \lor (A \land B) \} \]

\[ A \cdot B = (A \lor B) \land B \]
Example

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Interesting Points

• $A \circ B$ is a subset of $A$.
• If $C \subseteq D$; then $C \circ B \subseteq D \circ B$.
• $(A \circ B) \circ B = A \circ B$.
• $A$ is a subset of $A \bullet B$.
• If $C \subseteq D$; then $C \bullet B \subseteq D \bullet B$.
• $(A \bullet B) \bullet B = A \bullet B$. 
Hit-or-Miss Transform (HMT)

• To detect an object in an image:

• Basic Idea:
  • Use the object as se for erosion of $A$ and detect possible fits.
  • Use the neighborhood of the object as se for erosion of $A^c$ and find over fits.
  • Combine the two to detect exact fits.
HMT: Example

\[
A = X \cup Y \cup Z
\]

\[
W
\]

\[
(W - X)
\]

\[
A^c
\]

\[
A^c \ominus (W - X)
\]

\[
(A \ominus X) \cap (A^c \ominus (W - X))
\]

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Morphological Algorithms

- Boundary Extraction
- Region Filling
- Connected Components
- Convex Hull
- Thinning
- Thickening
- Skeletonization
- Pruning
Boundary Extraction

• Boundary of A is computed as:

\[ \beta(A) = A - (A \ominus B) \]
Boundary Extraction: Example
Region Filling

- Fills a region, whose boundary is given as 8-connected neighbours:

\[ X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \ldots \]